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Ballistic model of collisional absorption in strong laser fields

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Abstract
The ballistic model of collisional absorption in the presence of an intense laser field is exposed and the resulting collision frequencies are shown to be in good agreement with those obtained from the dielectric model. Advantages in favor of the ballistic model and shortcomings of both are discussed. Misleading physical interpretations from the dielectric model are shown to have their origin in a partially inadequate mathematical treatment.

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1. Introduction

In a plasma of ions of charge Z and density n_i a current of density $j = -en_e u$ (e electron charge, u mean electron velocity, $n_e = Zn_i$) is attenuated by the deflection of single electrons in the Coulomb field of single ions, and kinetic electron energy is converted into heat by friction. When an alternating field of frequency ω is applied, e.g. by irradiating the plasma by a laser under steady-state condition, the cycle-averaged quantity $\overline{\boldsymbol{jE}}$, $\boldsymbol{E}$ applied laser field represents the true dissipation. At a given time instant only part of the work $\boldsymbol{jE}$ goes into heat by friction, the remaining fraction leading to a change of the kinetic energy of the electron fluid.

In a laser field of arbitrary strength cycle-averaged absorption has been calculated, classically first in the so-called dielectric model (DM), i.e. electrons treated as a fluid, by Silin [1] and later by Dawson and Oberman [2], Jones and Lee [3] Decker *et al* [4] and others, and alternatively in the so-called ballistic model (BM, momentum loss of single electrons to screened ions) by Pert [5] and the present authors [6] and others. The numerous quantum treatments are mostly presented in the DM by Kremp *et al* and Bonitz *et al* (Kadanoff–Baym formalism [7]) and by Kull and Plagne (quantum Vlasov equation [8]); cycle-averaged ballistic treatments are presented, among others, by Shima and Yatom [9] and Silin and Uryupin [10]. Equivalently to $\overline{\boldsymbol{jE}}$ a cycle-averaged electron–ion collision frequency $\overline{\nu_{\rm ei}}$ can be introduced by the Drude ansatz

$$\frac{\mathrm{d}\boldsymbol{u}}{\mathrm{d}t} + \nu_{\mathrm{ei}}\boldsymbol{u} = -\frac{e}{m_e}\boldsymbol{E}, \tag{1}$$

where ν_{ei} is in general $\boldsymbol{E}$-dependent. By multiplying (1) by $\boldsymbol{u}$ and averaging over one cycle ω under steady-state conditions one arrives at $\overline{\nu_{\mathrm{ei}}} = \overline{\boldsymbol{jE}}/2n_e\mathcal{E}_{\mathrm{os}}$ with $\mathcal{E}_{\mathrm{os}}$ the mean oscillation energy of the single electron. In a wide range of parameters $\nu_{\mathrm{ei}} \ll \omega$ holds and hence, the oscillation velocity $\boldsymbol{u} = \boldsymbol{v}_{\mathrm{os}}$ can be assumed to be sinusoidal and $\mathcal{E}_{\mathrm{os}} = m_e\hat{v}_{\mathrm{os}}^2/4$, $\hat{v}_{\mathrm{os}}$ amplitude. In the DM the quantity $\overline{\boldsymbol{jE}}$ reads for an isotropic electron distribution function $f(\boldsymbol{v}_e)$ in terms of the Bessel functions J_l, plasma frequency ω_{p} and the quantum dielectric function $\epsilon_{\mathrm{q}}(k,\omega)$ as [7],

$$\overline{\boldsymbol{jE}} = \frac{Zm_e\omega_p^4}{\pi^2\hat{v}_{\mathrm{os}}}\int_0^\infty \frac{\mathrm{d}k}{k}F\left(\frac{k}{k_{\mathrm{D}}},\frac{\omega}{\omega_p},\frac{\hat{v}_{\mathrm{os}}}{v_{\mathrm{th}}}\right), \qquad F = \frac{\omega^2}{\omega_p^2}\sum_{l=1}^{\infty} l\frac{\Im\epsilon_{\mathrm{q}}(k,\mathrm{i}l\omega)}{|\epsilon_{\mathrm{q}}(k,\mathrm{i}l\omega)|^2}\int_0^{\frac{k\hat{v}_{\mathrm{os}}}{\omega}}\mathrm{d}\xi\, J_l^2(\xi). \tag{2}$$

Asymptotic formulae have been derived for a Maxwellian distribution function $f(\boldsymbol{v}_e) = f_{\mathrm{M}}(v_e)$ for small and large ratios $w = \hat{v}_{\mathrm{os}}/v_{\mathrm{th}}$, v_{th} mean thermal velocity:

$$w \ll 1: \overline{\nu}_{\mathrm{ei}} = \frac{4}{3}(2\pi)^{1/2}\left(\frac{Ze^2}{4\pi\epsilon_0 m_e}\right)\frac{1}{v_{\mathrm{th}}^3}\ln\Lambda_{\mathrm{S}}, \qquad \Lambda_{\mathrm{S}} = \frac{v_{\mathrm{th}}/\omega}{\lambda\!\!\!^{-}_{\mathrm{B}}}$$

$$w \gg 3: \text{Silin } \overline{\nu}_{\mathrm{ei}} = 2C\left(1 + \ln\frac{\hat{v}_{\mathrm{os}}}{2v_{\mathrm{th}}}\right)\ln\Lambda_{\mathrm{S}}, \qquad C = \frac{m_e\omega_{\mathrm{p}}^4}{\hat{v}_{\mathrm{os}}^3}$$

$$\text{Shima and Yatom } \overline{\nu}_{\mathrm{ei}} = C\ln\frac{2\hat{v}_{\mathrm{os}}}{v_{\mathrm{th}}}\left(\ln\Lambda_{\mathrm{S}} + \ln\frac{2\hat{v}_{\mathrm{os}}}{v_{\mathrm{th}}}\right) \tag{3}$$

$$\text{Kull and Plagne } \overline{\nu}_{\mathrm{ei}} = 2C\ln\frac{\hat{v}_{\mathrm{os}}}{v_{\mathrm{th}}}\ln(\sqrt{2}\Lambda_{\mathrm{S}}\sqrt{\hat{v}_{\mathrm{os}}/v_{\mathrm{th}}}).$$

Equation (2) looks rather complex. In particular the question of convergence arises. At large drift velocities $\hat{v}_{\mathrm{os}}$ up to 10^3 Bessel functions must be summed up to obtain an accurate result. To remove the differential operators $\partial/\partial t$ and nabla from the original kinetic equation transformation to Fourier space is done which, as a consequence, necessitates the decomposition of the anharmonic time dependence $\exp(\mathrm{i}\cos\omega t)$ into a complete set of Bessel functions. Loss of direct physical insight is the consequence. In addition, the underlying collision physics is obscured by the Fourier decomposition in k. In its nature a collision is short in comparison to the laser period ω. To decompose it into functions of $\boldsymbol{kx}$ of constant amplitude is inadequate. In transport theory time dependent quantities are of interest, e.g., heating function $\boldsymbol{j}(t)\boldsymbol{E}(t) - 1/2m_e n_e\,\mathrm{d}\boldsymbol{v}_{\mathrm{os}}^2/\mathrm{d}t$ at low frequency omega. It is a hopeless enterprise to evaluate it in the framework of the standard dielectric theory owing to the appearance of an infinite double sum of products J_lJ_n. Searching for a more appropriate description, i.e., a more adequate basis than plane waves may represent a challenging problem of mathematical physics. Expressions (3) differ from each other. What is the range of validity of the fits? The Spitzer–Silin-like Coulomb logarithm $\ln\Lambda_{\mathrm{S}}$ containing the thermal velocity and no drift as well as the double logarithm $\ln w\ln\Lambda_{\mathrm{S}} = \ln\left(\Lambda_{\mathrm{S}}^{\ln w}\right)$ needs explanation about their physical origin. The BM exposed in the following will represent a significant step into the right physical direction.

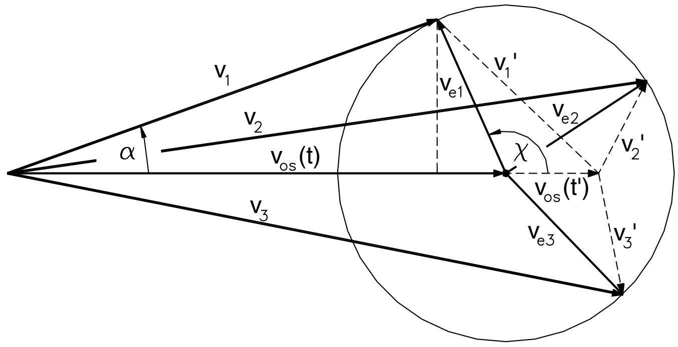

Figure 1. Composition of total velocity $\boldsymbol{v} = \boldsymbol{v}_{\rm os}(t)$, three examples. $\boldsymbol{v}_e$ taken from $f(\boldsymbol{v}_e)$.

2. The ballistic model

2.1. Momentum loss of a single electron

Collisions of individual electrons with a fixed ion of charge number Z are considered. The momentum loss along the original electron trajectory due to deflection is calculated and the result is averaged over all impact parameters b and all velocities $\boldsymbol{v}(t) = \boldsymbol{v}_{\rm os}(t) + \boldsymbol{v}_e$, $\boldsymbol{v}_e$ to be taken in the oscillating frame from a locally homogeneous and isotropic distribution function $f(\boldsymbol{v}_e) = f(v_e)$ for simplicity. As a result the time-dependent collision frequency $\nu_{\rm ei}(t)$ is obtained. Finally cycle averaging yields $\overline{\nu}_{\rm ei}$. The momentum loss $\Delta\boldsymbol{p}$ in a single Coulomb collision along $\boldsymbol{v}(t)$ follows from the differential Coulomb cross section $\sigma_\Omega = b_\perp^2 / 4\sin^4(\vartheta/2)$, $b_\perp$ collision parameter for perpendicular deflection, by integration over b, with σ the total cross section:

$$\Delta\boldsymbol{p} = m_e\boldsymbol{v}\frac{b_\perp^2}{\sigma}\int_{\vartheta=\epsilon}^{\pi}\frac{(1-\cos\vartheta)\sin\vartheta}{4\sin^4\frac{\vartheta}{2}}\,\mathrm{d}\vartheta\,\mathrm{d}\phi = 4\pi m_e\boldsymbol{v}\frac{b_\perp^2}{\sigma}\ln\frac{\left(b_\perp^2+b_{\rm max}^2\right)^{1/2}}{b_\perp}. \tag{4}$$

The Coulomb logarithm $\ln\left[\left(b_\perp^2+b_{\rm max}^2\right)^{1/2}/b_\perp\right]$ is fixed below. From (4) the momentum loss of an electron per unit time is $\dot{\boldsymbol{p}} = -m_e\nu_{\rm ei}(\boldsymbol{v})\boldsymbol{v} = -K/v^3\boldsymbol{v}\ln\Lambda$, $K = Z^2e^4n_i/\left(4\pi\varepsilon_0^2m_e\right)$. The velocity is $v = |\boldsymbol{v}| = \left(\boldsymbol{v}_{\rm os}^2+\boldsymbol{v}_e^2+2|\boldsymbol{v}_{\rm os}||\boldsymbol{v}_e|\cos\chi\right)^{1/2}$ (see figure 1); $\nu_{\rm ei}$ follows as

$$\nu_{\rm ei}(t) = 2\pi\frac{K}{m_e v_{\rm os}(t)}\int_0^\infty\int_{-1}^{+1}\frac{\cos\chi\, v_e^2 f(v_e)\ln\Lambda}{v_{\rm os}^2+v_e^2+2v_{\rm os}v_e\cos\chi}\,\mathrm{d}\cos\chi\,\mathrm{d}v_e, \tag{5}$$

$$\overline{\nu}_{\rm ei} = 4\pi\frac{K}{m_e\hat{v}_{\rm os}^2}\overline{\int_0^\infty\int_{-1}^{+1}\frac{\boldsymbol{v}_{\rm os}\boldsymbol{v}}{v^3}v_e^2f(v_e)\ln\Lambda\,\mathrm{d}\cos\chi\,\mathrm{d}v_e}. \tag{6}$$

Expressions (5) and (6) are much simpler than (2), however they do not handle (yet) collective resonances around $\omega = \omega_{\rm p}$ [2, 7, 8]. The integration over b in $\ln\Lambda$ runs from $b = 0$ to $b = b_{\rm max}$. No lower cutoff or 'distance of the closest approach' appears; only $b_{\rm max}$ must be determined.

2.2. Cutoffs

Let us assume that $b_\perp$ and $\lambda_{\rm B} = \hbar/m_e v$ are well separated from $b_{\rm max}$ and the Debye length $\lambda_{\rm D}$; in addition $b_{\rm max} \gg b_0$ separating straight from bent orbits is assumed to be valid, as is the

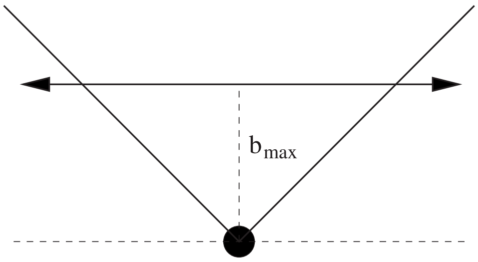

Figure 2. Jackson's model of Coulomb interaction. The particle feels the constant maximum attraction over the distance $2b$ and zero force outside. It reproduces the Coulomb cross section.

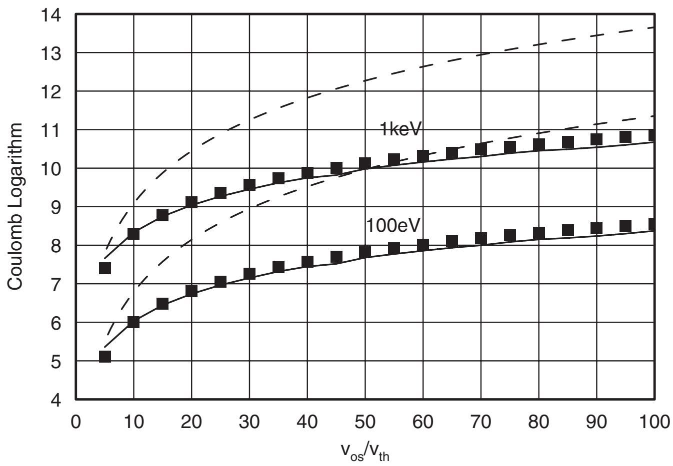

Figure 3. $\overline{\nu}_{ei}$ from DM (solid), BM (squares), dashed lines from (9). $n_e = 10^{21}$ cm^{-3}, $\omega = 5\omega_p$.

case in the weakly coupled plasma. First b_{max} is fixed. Under the current conditions Jackson's model applies [11], see figure 2. For $\hat{v}_{os}/v_{th} \gtrsim 3$ and $\omega > \omega_p$ the time-averaged speed is, owing to $v^2 \sim \sin^2 \omega t$, $\overline{v} = v/\sqrt{2}$. From $\tau_{int} = 2b/v \leqslant \tau_{Laser}/3$ follows

$$\frac{2b_{max}}{v/\sqrt{2}} = \frac{1}{3}\frac{2\pi}{\omega} \Longrightarrow b_{max} = \frac{v}{\sqrt{2}\omega}. \tag{7}$$

The accurateness of this cutoff can be checked by comparing (6) with (2). The outcome will result very satisfactory (see figure 3). Under the conditions above and $w \ll 1$ the differential cross section in the first Born approximation from the shielded Debye potential applies with the result $b_\perp$ in $\ln \Lambda$ to be replaced by $b_{min} = \lambda_B/2$, if this quantity is larger than $b_\perp$. In the opposite case, occurring preferentially at low electron temperature the situation is more complex. As λ_B depends on the individual particle velocity v, $b_{min} = \hbar/2m_e v$ is of general validity [12]. Thus,

$$\ln \Lambda = \ln \left(b_{max}^2 + b_\perp^2\right)^{1/2}/b_{min}, \qquad b_{max} = v/\sqrt{2}\max(\omega, \omega_p), \qquad b_{min} = \hbar/2m_e v. \tag{8}$$

In the literature $b = b_\perp$ or $b = \lambda_B$ is generally referred to as a 'lower cutoff' b_{min}. This is unfortunate and totally misleading.

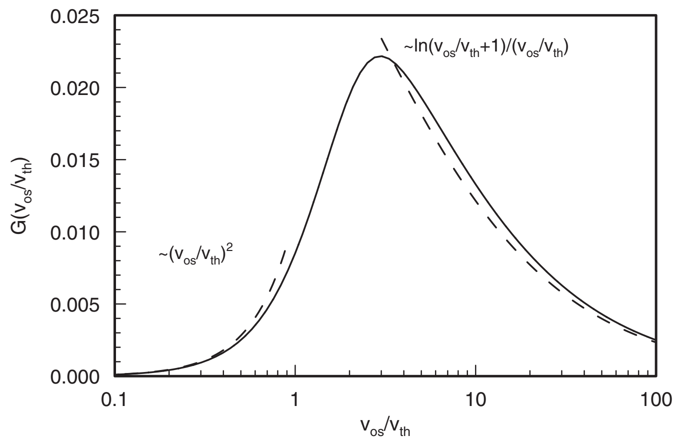

Figure 4. The function $G(\hat{v}_{\rm os}/v_{\rm th})$ and two asymptotes, dashed.

2.3. Averaging procedure for Maxwellian $f_{\rm M}(v_e)$

$\overline{\nu}_{\rm ei}$ from (6) is evaluated for $n_e = 10^{21}$ cm^{-3}, $\omega = 5\omega_{\rm p}$ and $T_e = 100$ eV and $T_e = 1$ keV with $\ln\Lambda$ from (8), and is compared with $\overline{\nu}_{\rm ei}$ from (2) (figure 3). There is very good agreement. For a rough estimate $\ln\Lambda$ may be replaced by the average $\ln\langle\overline{\Lambda}\rangle = \ln\left(\hat{v}_{\rm os}^2/4 + v_{\rm th}^2\right)^{1/2}/b_{\rm min}$ and treated as a constant, and $\cos\alpha = 1$ may be set, α from figure 1. Then $\overline{\nu}_{\rm ei}$ simplifies to

$$\overline{\nu}_{\rm ei} = \frac{K\ln\langle\overline{\Lambda}\rangle}{m_e\hat{v}_{\rm os}^2}\overline{\frac{1}{v_{\rm os}(t)}\int_0^{v_{\rm os}(t)} 4\pi v_e^2 f_{\rm M}(v_e)\,\mathrm{d}v_e} = \frac{K\ln\langle\overline{\Lambda}\rangle}{m_e\hat{v}_{\rm os}^2}G(\hat{v}_{\rm os}/v_{\rm th}). \tag{9}$$

Its evaluation for the parameters of figure 3 yields the two dashed lines. They are by 25–30% too large. It is instructive to replace $\ln\langle\overline{\Lambda}\rangle$ in (9) by the correct value $\langle\overline{\ln\Lambda}\rangle$ which is accomplished by equating (9) and (6). The function $G(\hat{v}_{\rm os}/v_{\rm th})$ is shown in figure 4. There are also shown the asymptotes proportional to $(\hat{v}_{\rm os}/v_{\rm th})^2$ for $\hat{v}_{\rm os} < v_{\rm th}$ and $\ln(\hat{v}_{\rm os}/v_{\rm th}+1)/(\hat{v}_{\rm os}/v_{\rm th})$ for $\hat{v}_{\rm os}/v_{\rm th} \gtrsim 3$.

3. Discussion and conclusion

The BM has several advantages in comparison to the various versions of the DM. As figure 3 shows it yields collision frequencies which are nearly identical with the DM values. The numerical effort to calculate them is by far simpler and quicker. Test calculations show the same or slightly higher agreement also for laser frequencies $\omega = 0.5\omega_{\rm p}$ and $\omega = 1.5\omega_{\rm p}$. The simplified version (9) yields analytical formulae in a wide parameter range [6], however not without loss of exactness (25–30% too large). The next great advantage of the BM is its physical description of collision events. The extension to strongly coupled plasmas, i.e such for which $b_\perp$ becomes of the same order of $\lambda_{\rm D}$, however $b_\perp < n_e^{-1/3}$ is straightforward in the BM. In the DM large angle deflections are not included and must be treated separately. Various aspects that appear paradoxical in the DM find their natural explanation in the BM. For example, with the natural cutoffs (8) (where only $b_{\rm max}$ is a real cutoff) and the results of figures 3 and 4 show that the double logarithms and $\ln\Lambda_{\rm s}$ in the asymptotic expressions (3) are merely mathematical approximations without any relationship with physics; the continuous line in figure 4 may be fitted more exactly by approximations of different mathematical structures. The term $\ln(\hat{v}_{os}/v_{th})$ is not produced by shielding. As the dielectric function $\epsilon(k, \omega)$ contains $\lambda_D = v/\omega_p$ and the correct expression for b_{max} stems from large k values in the upper limit of the Bessel integrals in (2) one could be tempted to claim that the 'long wavelength approximation' $k\hat{v}_{os}/\omega \lesssim 1$ is wrong. However, the BM shows that it is correct at all frequencies ω. It shows once more that a Fourier decomposition for collisional events is physically inappropriate. Two clear advantages of the DM are the correct description of shielding in weakly coupled plasmas and of resonances in $\overline{\nu}_{ei}(\omega)$ around ω_p [2, 7, 8].

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